# COURSE: PHYSICS (H)

# SEMESTER: VI

# Some topics of

# PHY-DSE-4.T NUCLEAR & PARTICLE PHYICS


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# Nuclear Models.

To understand and describe the nuclear structure, physicists have developed several models. Each model succeeds only in explaining a portion of the nuclear experimental results but fails in other. The failure of one model is the success of another and therefore they collectively are capable of explaining nearly all the results.

## The Liquid Drop Model.

The liquid-drop model treats the nucleons as molecules in a drop of a liquid and the nucleus is considered to be a homogeneous mixture of nucleons in which all the nucleons interact strongly with each other. As a result, the internal energy of the nucleus is equally distributed among the constituent nucleons, while surface tension forces tend to keep the molecules spherical. This is analogous to a drop of liquid and hence model is called liquid drop model of the nucleus.

## Similarities between a nucleus and a liquid drop/ assumptions in a liquid drop model.

The liquid drop model is based on the following simplifying assumptions:


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1. In equilibrium state, both liquid drop and nucleus are spherical in shape

2. Nuclear forces in a nucleus are short range and saturated forces. Similarly, intermolecular forces in a liquid drop are short range and saturated forces.

3. The density of a liquid drop is independent of the size of the drop. Similarly, the density of the nucleus is independent of size of the nucleus.

4. In a nucleus the binding energy per nucleon is constant. Similarly, in a liquid latent heat of vaporization is constant.

5. A thermally agitated liquid drop may release its extra energy by cooling, i.e by radiating heat away. Similarly, an excited nucleus de-excites by emitting gamma radiations.

6. The evaporation of a liquid is similar to the emission of radiations ($\alpha, \beta, \gamma$...) by radioactive nuclei.

7. A liquid drop may split into two smaller drops. Similarly, an unstable large nucleus may break up into two smaller nuclei by the


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process called nuclear fission.

8. Two small liquid drops can fuse into a bigger drop. Likewise two small nuclei can fuse into a bigger nucleus by the process called nuclear ~~fission~~ fusion

The nucleus is regarded as a liquid drop with nucleons playing the role of molecules.

Differences between a nucleus and a liquid drop.

1. The nuclear density is constant for any type of nucleus. However, the density of one type of liquid is different from that of the other type of liquid.

2. A much larger fraction of nucleons is on the surface relative to the bulk of nucleons inside the nucleus as compared to the smaller fraction of molecules on the surface relative to the bulk of molecules within a liquid drop.

3. The nucleus is a charged body and obeys the laws of quantum mechanics. However, a liquid drop is a neutral classical system.

4. Shell structure as evident in nucleus is not


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observed in a liquid drop.

5. Only two types of particles, neutrons and protons from all the nuclei. On the other side, there is a large number of different types of molecules forming different types of liquids.

## The semi-empirical mass (Bethe-Weizsäcker) formula

The semi-empirical mass formula, developed by Hans Bethe and von Weizsäcker in 1935 is based on theoretical aspects as well as empirical (experimental) data of nuclei. At first, the semi-empirical mass formula was developed purely using the quantitative aspects of liquid drop model and later it was modified by incorporating some quantum mechanical treatment.

### Assumptions:

The semi-empirical mass formula was developed by using the following assumptions:

1. Nuclear density is Constant
2. Neutrons and protons, being fermions, occupy energy levels symmetrically and separately
3. The nuclear forces are charge independent
4. The nuclear forces are short range and Saturated forces.


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5. According to the laws of quantum mechanics pairing effect of nucleons take place.

The mass of the nucleus is given by

$$M(A,Z) = Z \times m_p + N m_n - \frac{E_B}{c^2}$$

$$\Rightarrow M(A,Z) = Z m_p + (A-Z) m_n - \frac{E_B}{c^2} \quad \text{— (1)}$$

According to the semi-empirical mass formula, the binding energy $E_B$ of a nucleus is treated as a sum of five terms as given below:

(i) Volume energy term:

The volume energy term accounts for the fact that total binding energy $E_B$ is approximately linearly proportional to number of nucleons A ie

$$E_B \propto A$$

Therefore the volume energy term is written as:

$$E_V = a_V A \quad \text{— (2)}$$

Where $a_V$ (in MeV) is the constant of proportionality to be determined empirically. $E_V$ is called volume energy as nuclear volume is proportional to A.


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Explanation :

(a) Nuclear forces binding the nucleons together are short range and saturated. If it was not so each nucleon would interact with the remaining $(A-1)$ nucleons and the total number of interactions would be $A(A-1) \approx A^2$ (for large A). But binding energy is proportional to A and not $A^2$. This indicates each nucleon interacts only with its nearest nucleons.

(b) This term is +ve as nuclear force is attractive producing positive contribution to binding energy of the nucleus.

(ii) Surface energy term $E_S$ :

The surface energy term accounts for the fact that binding energy lost is proportional to the surface area.

Explanation:

(a) A nucleon at the surface of a nucleus interacts with fewer other nucleons and hence has lesser binding energy in comparision to a nucleon in the interior of the nucleus as shown in fig 1

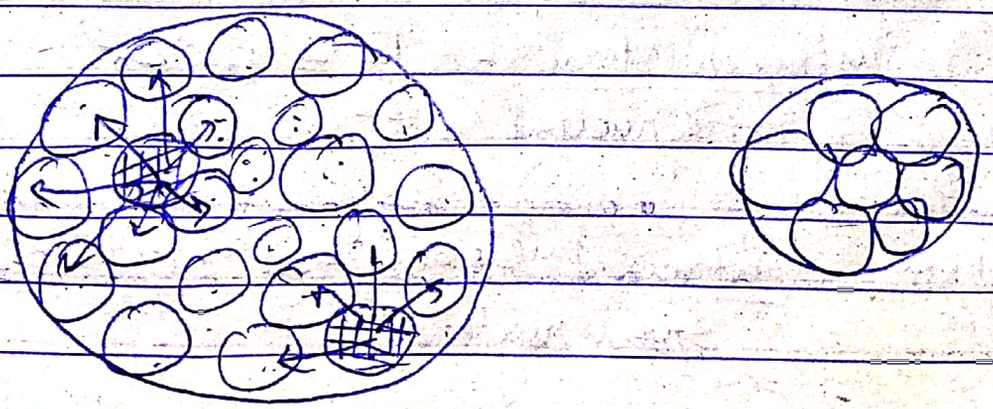


fig 1 Illustration of attraction of surface nucleons and nucleons well inside the nucleus.



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Therefore, in the volume energy term where it is assumed that each nucleon in the nucleus contributes equally to the binding energy, over estimates the binding energy for surface nucleons. The over estimated amount of energy, so called surface energy $E_s$, is proportional to the number of nucleons at the surface, which in turn is proportional to the surface area $(4\pi R^2)$ of the nucleus

Thus $E_s \propto 4\pi R^2$

$\propto 4\pi R_0^2 A^{2/3}$ $\quad [\because R = r_0 A^{1/3}$

here $4\pi R_0^2$ is a constant which when multiplied with the constant of proportionality gives another constant.

$$E_s = -a_s A^{2/3} \quad \text{---} (3)$$

(b) This term is -ve as this over estimated term has to be substracted from the volume energy term.

(c) The fraction of nucleus on the surface is large for light nuclei than for heavy nuclei, hence light nuclei have large surface energy and less binding energy.

For large one: $\frac{\text{Surface nucleons}}{\text{Total nucleons}} = \frac{12}{22} = 0.545$

Smaller (light) one $\frac{\text{Surface nucleons}}{\text{Total nucleons}} = \frac{6}{7} = 0.857$


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(iii) Coulomb energy term:

The Coulomb repulsion between the protons in the nucleus also tend to weaken the nuclear binding. The repulsive force being of long range type can act between all the proton pairs in the nucleus. The no. of such pairs is $Z(Z-1)/2 \approx Z^2/2$ for Z large.

The Coulomb potential energy for all the protons within the nucleus can be calculated as follows:

If we assume the total nuclear charge $Q = +Ze$ to be distributed uniformly then the charge density will be

$$\rho_e = \frac{Ze}{\frac{4}{3}\pi R^3} = \frac{3Ze}{4\pi R^3} = \frac{3Ze}{4\pi r_0^3 A} \quad \text{---(1)}$$

The potential energy of the charged sphere of radius R can be calculated by building up the sphere, layer by layer, making use of the fact that field outside a spherical distribution of charge is same as if all the charge is ~~distributed~~ concentrated at the centre. Suppose the sphere has been built up to a radius r. The total charge in it

$$q = \frac{4}{3}\pi r^3 \rho_e$$


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Let us now add an infinitesimal shell of charge of thickness dr to this. The work done in bringing the charge $dq = 4\pi r^2 \rho_c dr$ in the shell from infinity to the radius r against the field of charged sphere of radius r, as above is

$$dw = \frac{q\,dq}{4\pi\epsilon_0 r} = \frac{4}{3}\pi r^3 \rho_c \times \frac{4\pi r^2 \rho_c dr}{4\pi\epsilon_0 r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{16\pi^2 \rho_c^2 r^4 dr}{3}$$

Hence the total work done in forming the charged sphere of radius R is

$$W = \frac{1}{4\pi\epsilon_0} \frac{16\pi^2 \rho_c^2}{3} \int_0^R r^4 dr$$

$$= \frac{1}{4\pi\epsilon_0} \frac{16\pi^2 \rho_c^2}{3} \frac{R^5}{5}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{16\pi^2}{3} \left(\frac{3Q}{4\pi R^3}\right)^2 \frac{R^5}{5} \quad \left[\text{Since } \rho_c = \frac{Q}{\frac{4}{3}\pi R^3}\right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{16\pi^2}{3} \times \frac{3\times 3\times Q^2}{4\pi \times 4\pi \times R^6} \times \frac{R^5}{5}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{3}{5} \frac{Q^2}{R}$$


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Since the potential energy is negative of the work done, the Coulomb energy of the charged sphere is

$$E_C = -\frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R} = -\frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 R}$$

$$= -\frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 r_0 A^{1/3}}$$

$$E_C = -a_c \frac{Z^2}{A^{1/3}} \quad \text{— (5)}$$

Where $a_c = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 r_0}$ — (6)

Explanation:

(a) Due to the $Z^2$-dependence, this term becomes large for some very high Z nuclei beyond which it starts overcoming the total binding energy. Therefore such nuclei are never formed.

(b) This term is negative because Coulomb repulsive energy acts negatively towards the binding energy of the nucleus.

Note: The first three terms for the binding energy of a nucleus arise from purely classical considerations. Earlier the semi-empirical formula


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included only these three terms which could not produce the binding energy curve to the desired accuracy. Therefore to refine the formula quantum behaviour of nucleons was taken into consideration, which resulted to the following two terms.

(iv) Asymmetry energy term $E_a$:

Assymetry energy term, which has no basis in the liquid drop model is purely quantum mechanical term and accounts for the preference for $N = Z$, giving the most strongly bound nuclei for given A. The origin of the asymmetry term is the Pauli exclusion principle that forbids the fermions to occupy the same quantum state.

Explanation:

(a) In heavy nuclei the repulsive Coulomb force ($\propto Z^2$) increases more rapidly than the attractive nuclear force ($\propto A$). Therefore it is necessary for heavy nuclei to have $N > Z$, so that Coulomb force may not overcome the nuclear force, otherwise such nuclei can not be formed.


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The deficient energy also called the asymmetry energy Ea, is originated from the the fact that protons and neutrons occupy discrete energy levels (independently) to obey the Pauli exclusion principle (fig). In N>Z case neutrons levels end higher than proton levels which affect binding energy in the following two ways.

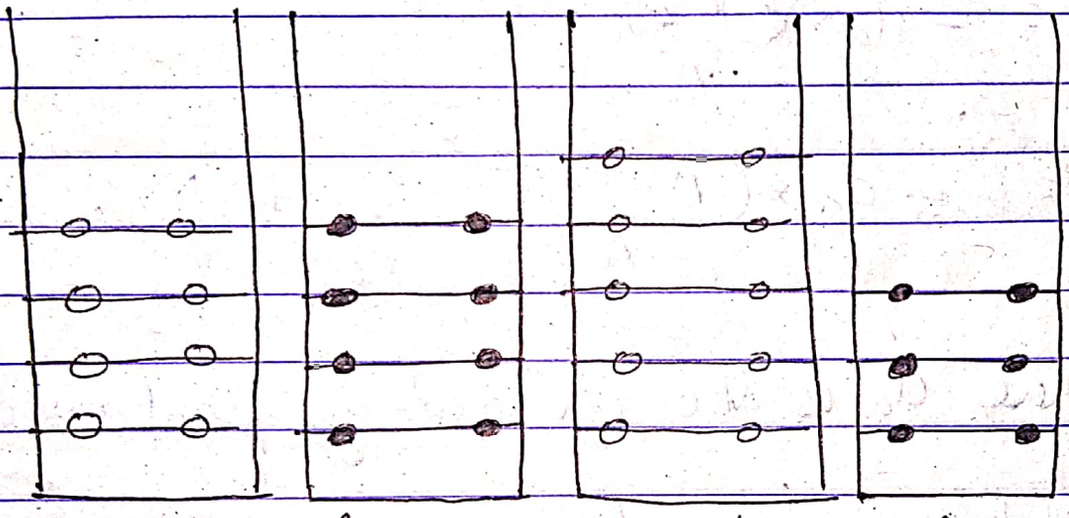



fig Separate energy levels for neutrons and protons

(I) The excess N-Z neutrons are out of reach of other nucleons and hence have less binding energy than the nucleons occupied in the deeper energy levels.

(II) The departure from symmetry causes a change in the nuclear volume by a


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fraction of $(N-Z)/A$ and hence affects binding energy due to volume energy term.

The asymmetry energy proportional to these two factors can be expressed as:

$$E_a = -a_a \times \frac{(N-Z)(N-Z)}{A}$$

$$= -a_a \times \frac{(N-Z)^2}{A}$$

or $$E_a = -a_a \times \frac{(A-2Z)^2}{A} \quad \because N = A-Z \quad \text{— (7)}$$

Where $a_a$ is the Constant of proportionality

(b) This term is -ve because the asymmetry term also decreases the binding energy.

(v) Pairing energy term $E_p$:

The last term called the pairing energy term and arise from the tendency of the nucleons to form pairs (p-p and n-n) in order to increase the stability and hence the binding energy of the Nucleus. Pairing of nucleons take place for two nucleons in the same energy level such that their spin and orbital motions are


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opposite to each other.

Explanation:

A close study of the binding energies of different nucleus shows that for a given A (even), the even Z - even N (e-e) nuclei are more strongly bound than the odd Z - odd N nuclei (o-o) nuclei. Further for odd A nuclei, both even Z - odd N (e-o) and odd Z - even N (o-e), it is found that the bindings of odd A nuclei are intermediate between e-e and o-o nuclei. The odd A nuclei are more strongly bound than o-o nuclei while they are less strongly bound than e-e nuclei.

In the expression for $E_B$, pairing term $E_p$ is added, which arise due to pairing of nucleons of same type with opposite spins. Such pairing tends to increase the strength of binding which thus becomes maximum for e-e nuclei in which all nucleons of both type (p and n) are paired with oppositely aligned spins. In odd A nuclei, there is one unpaired nucleon which is a neutron for (e-o) and proton for (o-e) nuclei. This weakens the binding in such nuclei by about 2-3 MeV. Finally in o-o nuclei there are two unpaired nucleons one proton and one neutron. As a result the


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binding of such nuclei are further weakened by about 2 to 3 MeV.

The pairing energy term is given by $E_p = \pm \delta$, $\delta$ depends on A only and is taken to be zero for odd A nuclei, positive for e-e nuclei and negative for o-o nuclei. It is given by

$$\delta = a_p A^{-3/4} \quad — (8)$$

We can thus write the binding energy as

$$E_B(A,Z) = E_V - E_S - E_C - E_a \pm \delta$$

$$\text{or} \quad E_B = a_v A - a_s A^{2/3} - a_c Z^2/A^{1/3} - a_a \frac{(A-2Z)^2}{A} \pm \delta \quad — (9)$$

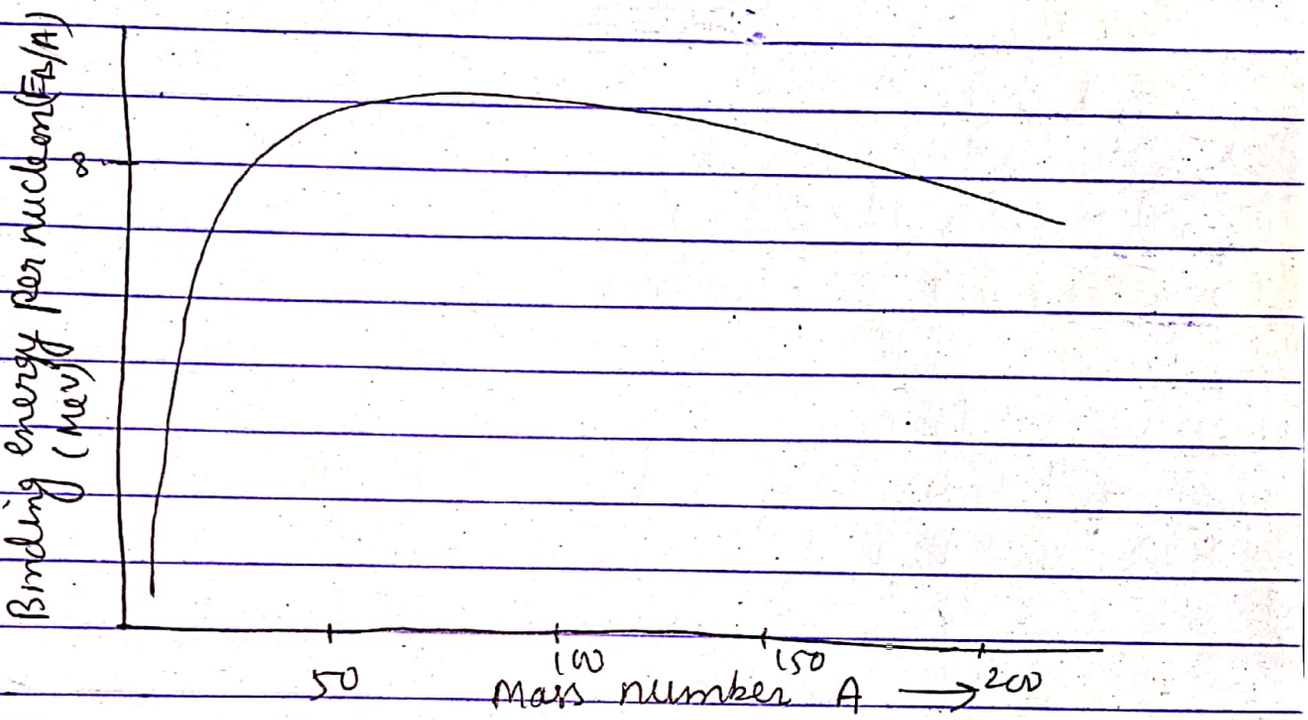
Substituting this value of $E_B$ from (9) to eqn (1)

$$M(A,Z) - Z \times m_p + (A-Z) m_n - \frac{1}{c^2}\left[a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a (A-2Z) A^{-1} \pm \delta\right] \quad — (10)$$

The above equation is known as semi empirical mass formula or Bethe-Weizsäcker semi empirical mass formula.


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The major success of the formula lies in the fact that the binding energy curve drawn from the formula approximately matches the experimental binding energy curve.




Binding energy per nucleon versus mass no. A.

It is empirical in the sense that the constants ($a_v$, $a_s$, $a_c$, $a_a$ and $a_p$) have to be determined experiment-ally.


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# Nuclear shell model

In the absence of exact knowledge of nucleus structure (due to lack of knowledge of forces between nucleas particles) various nuclear models have been proposed to explain the observed nuclear properties and various phenomenon exhibited by nuclei.

Each nuclear model makes certain simplifying assumptions and is aimed to explain only a given set of properties and a given set of phenomenon. In the early stage of its development one of the main objectives of the shell model was to reproduce the so called magic numbers. It is observed that all such nuclei which have neutron no. & proton no. & both


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equal to any one of the following nos 2, 8, 20, 28, 50, 82 and 126. Exhibit characteristics properties. It was found that this model was capable of explaining not only the magic nos but also many other nuclear property's such as, spin, magnetic moment and energy levels.

As in the case of electron shells in atoms, it was proposed that nucleons in a nucleus exists in different shells or orbits called nuclear shells. In order to get the concept of nuclear shells the fundamental assumption was made that any nucleon in a nucleus moves under the action of a central force field created by the rest of the nucleons. In this model the motion of the nucleons are treated independently of each other. Thus, in effect means that the interaction between nucleons in a nucleus is weak.

To describe determine how the nucleons fill the various quantum states of the nucleus, it is necessary to specify the mean potential to which each nucleon is subjected. Very strong experimental evidence exists which supports the point of view ~~when~~ that when a nucleus contains 2, 8, 20, 50, 82 & 126 protons, the so called 'magic nos' - a shell closure ~~&~~ occurs ie the energy level just above the shell closure level is separated by a larger energy ~~level~~ gap first spacing than those within the shell. To understand this shell closure property and to predict electromagnetic and nuclear ground state properties of the nucleus in terms of the


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uncorrelated motion of single particles in the specified mean potential we consider the level scheme predicted by (i) the infinite square well potential and (ii) the infinite harmonic oscillator potential.

In the CM system the Schroedinger eqn describing the motion of nucleon in the nucleus satisfied by the wave function describing the state of the nucleon can be written as

$$\nabla^2\psi + \frac{2m}{\hbar^2}[E - V(r)]\psi = 0 \quad \text{—(1)}$$

where $m$ is the reduced mass which is practically equal to mass of the nucleon in the case of a moderating heavy and heavier nucleus.

$V(r)$ is the spherically symmetric potential offered to the nucleon by the rest of the nucleons in the nucleus.

The most general solution of this eqn in spherical polar coordinates is given by

$$\psi_{n,l,m}(r,\theta,\phi) = U_{nl}(r)Y_{lm}(\theta,\phi)$$

where $U_{nl}(r)$ is the radial fun$^n$ and $Y_{lm}(\theta,\phi)$ is a spherical harmonic.

The set of quantum nos defines an eigen state for the nucleon. The radial function $U_{nl}(r)$ corresponds to an energy eigen value $E_{nl}$ satisfies the eq$^n$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU_{nl}}{dr}\right) + \frac{2m}{\hbar^2}\left[E_{nl} - V(r) - \frac{l(l+1)\hbar^2}{2mr^2}\right]U_{nl} = 0 \quad \text{—(2)}$$


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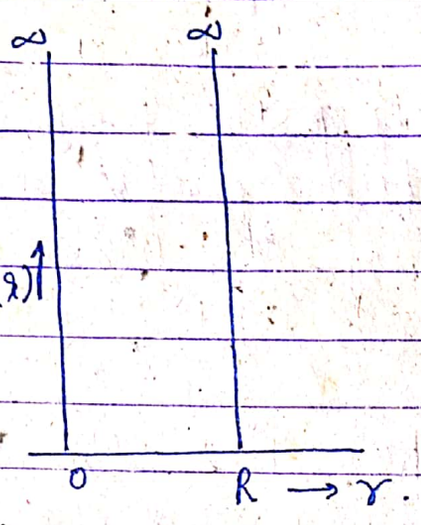
Now we Consider Infinite Square Well potential

We shall Consider the potential field in which the nucleon moves to be in an infinite Square well. In order to get the Exact Solution for $U_{nl}$ and $E_{nl}$ The potential fun Can be written as.

$V(r) = 0$ for $0 < r \leq R$

$= \infty$ for $r \leq 0$

$= \infty$ for $r \geq R$.




then we have

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU_{nl}}{dr}\right) + \frac{2m}{\hbar^2}\left[E_{nl} - \frac{l(l+1)\hbar^2}{2mr^2}\right]U_{nl} = 0$$

within the well

and $U_{nl}(r) = 0$ outside the well.

The solution within the well are the Spherical Bessel functions i.e.

$$U_{nl}(r) = J_l(K_{nl}\, r)$$

where $K_{nl} = \sqrt{\frac{2m}{\hbar^2}E_{nl}}$

We have the boundary Condition, in simple case of a well of infinite depth the wave function has to vanish at the nuclear boundary. Thus at $r = R$

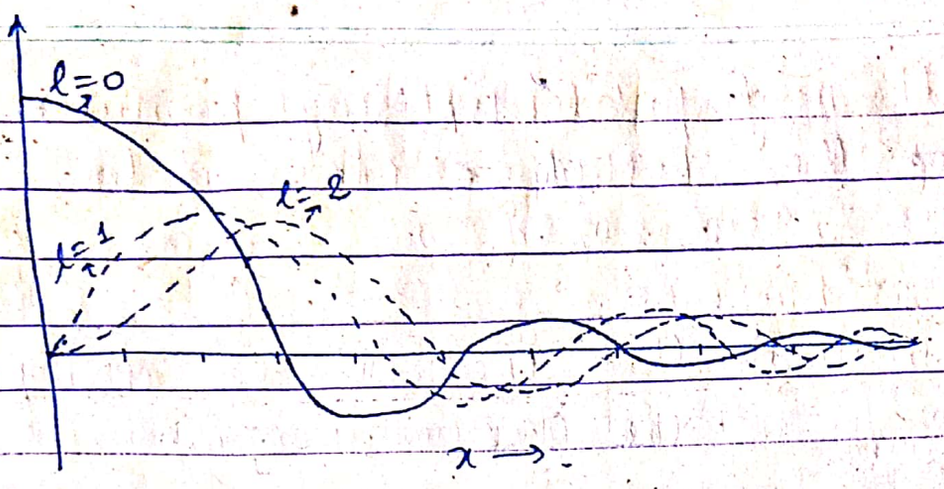
$$U_{nl}(R) = J_l(K_{nl}R) = 0$$

Each $l$-value has a set of zeros (roots) and each of them Corresponds to a value of $K_{nl}$ given by $K_{nl}R = x$.

The plot of $J_l(x)$ against $x$ for different values of $l$ are as shown in fig.



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From the graph we can find the values of $x$ i.e the values of $R$ne$R$ for which $J_l(x) = 0$

$R$ne$R = x$, $J_l(x) = 0$

40 6 2p ——————

34 14 1f ——————

20 2 2s —————— $x = x_4$

~~28~~ 10 1d —————— $x = x_3$

8 6 1p —————— $x = x_2$

2 2 1s —————— $x = x_1$

Each shell model level actually consists of $2l+1$ sublevel states. Each with a different angular wavefunction $Y_{l,m}(\theta, \phi)$ Because of the two different orientations of the nuclear spin, a level can have $2(2l+1)$ protons and since neutrons and protons are two different particles hence the level can also have $2(2l+1)$ neutrons in addition to $2(2l+1)$ protons.

The $l=0$ state is written as 1s, it consists $2(2.0+1) = 2$ particles. Similarly $l=1$ consists of 6 particles and so on. In the figure the extreme left hand side.


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represents the no of particles upto any particular level. Here we see the shell closure closes at total particle nos 2, 8, 18, 20, 34, 40 .... These are not the nuclear magic nos.

So we conclude that the infinite SQ well potential is unable to predict the observed magic nos.

Now let us consider another extreme kind of potential namely a harmonic oscillator potential. Such a potential is spherically symmetric & can be represented as

$$V(r) = \int_0^r kr\,dr = \frac{1}{2}kr^2 = \frac{1}{2}m\omega^2 r^2 \quad \text{— (3)}$$

The energy eigen values for a nucleon under the action of the harmonic oscillator potential given above are

$$E_{nl} = \hbar\omega\left(2n + l - \frac{1}{2}\right)$$

Let us define an integer $\Lambda$ as $\Lambda = 2n + l - 2$.

$$E_\Lambda = \hbar\omega\left(2n + l - 2 + 2 - \frac{1}{2}\right) = \hbar\omega\left(\Lambda + \frac{3}{2}\right)$$

With $n = 1, 2, 3, \cdots$ $l = 0, 1, 2, \cdots$ and $\Lambda = 2n + l - 2$.

When the angular dependence of the wavefunctions is examined it is found that for each $\Lambda$ value there is a degenerate group of levels with different $l$ values such that $0 \le l \le \Lambda$ and even (odd) corresponds to even (odd) values of $\Lambda$. Thus the sequence of single particle levels for harmonic oscillator well consists of bands of degenerate levels, each band separated by energy $\hbar\omega$ from the next. The degeneracy corresponding to each $l$ value is $2(2l+1)$.


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The single particle level scheme predicted by the infinite harmonic oscillator well is given by

| Λ | $E_\Lambda$ | n | l | states | nos. nucleons accomodated $2(2l+1)$ | | shell closure |
|---|---|---|---|---|---|---|---|
| 0 | $\frac{3}{2}\hbar\omega$ | 1 | 0 | 1s | 2 | | 2 |
| 1 | $5/2\hbar\omega$ | 1 | 1 | 1p | 6 | | 8 |
| 2 | $7/2\hbar\omega$ | 1 | 2 | 1d | 10 | 12 | 20 |
| | | 2 | 0 | 2s | 2 | | |
| 3 | $9/2\hbar\omega$ | 1 | 3 | 1f | 14 | 20 | 40 |
| | | 2 | 1 | 2p | 6 | | |
| 4 | $11/2\hbar\omega$ | 1 | 4 | 1g | 18 | 30 | 70 |
| | | 2 | 2 | 2d | 10 | | |
| | | 3 | 0 | 3s | 2 | | |

We observe the following

(i) The energy levels are equispaced. The spacing between successive energy level is $\hbar\omega$.

(ii) Unlike in the case of infinite square well potential the higher energy levels are degenerate.

(iii) Not all the shell closure no. are the observed magic nos.

Several attempts were made to modify the potential field in which a nucleon in a nucleus moves


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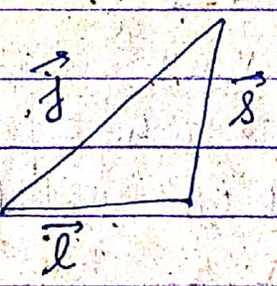
but in none of the cases the closed shell structures were found to be governed by observed magic nos.

In 1949 Mayer & Haxel and Jensen & Suess independently suggested that besides the central potential there must be a non-central potential in which a nucleon in a nucleus moves. They suggested that for hitherto unknown reason there occurs a strong spin orbit coupling ~~in a~~ of nucleon in a nucleus. The spin angular momentum and the orbital angular momentum of a nucleon combine to give a total angular momentum to the nucleon. The interaction corresponding to such coupling gives rise to non central potential. The spin orbit interaction energy as suggested by them was of the form $W = -f(r)\,\vec{s}\cdot\vec{l}$ —(4)

where $f(r)$ is a distance dependent potential and $\vec{s}, \vec{l}$ are respectively the spin angular momentum vector of and orbital angular momentum vector of nucleon. Thus the total potential in which a nucleon in a nucleus moves is $V = V(r) - f(r)\,\vec{s}\cdot\vec{l}$. —(5)

The spin orbit potential acts as a perturbation term and is expected that it will split the degeneracy of the energy level observed ~~only~~ ~~when~~ when only central potential is considered.

$\vec{l}$ and $\vec{s}$ combine to give $\vec{j}$ as shown




we then get according to cosine law in a triangle

$$j^2 = l^2 + s^2 + 2\vec{l}\cdot\vec{s}$$


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$$\vec{l}\cdot\vec{s} = \frac{j^2 - l^2 - s^2}{2}$$

$$\text{or } \vec{l}\cdot\vec{s} = \frac{j(j+1) - l(l+1) - s(s+1)}{2}$$

Since Spin of a nucleon is 1/2, there are only two possible ways $\vec{l}$ and $\vec{s}$ may combine.

i) $j = l + \frac{1}{2}$ → stretch case

ii) $j = l - \frac{1}{2}$ → Jackknife case.

We then get $(\vec{l}\cdot\vec{s})_{sb} = \frac{(l+\frac{1}{2})(l+\frac{3}{2}) - l(l+1) - \frac{1}{2}(\frac{3}{2})}{2}$

$$= l/2$$

And $(\vec{l}\cdot\vec{s})_{jkn} = \frac{(l-\frac{1}{2})(l+\frac{1}{2}) - l(l+1) - \frac{1}{2}\,3/2}{2}$.

$$= -\frac{(l+1)}{2}$$

Thus we get

$$(W)_{sb} = -f(r)\,\vec{s}\cdot\vec{l} = -\frac{l}{2}f(r) \quad -(6)$$

$$(W)_{jkn} = \frac{(l+1)}{2}f(r). \quad -(7)$$

The change of the splitting of an energy level from its central value are in the two cases given by

$$(\Delta E_{nl})_{sb} = \int \psi_{nl}^{*}\left[-\frac{l}{2}f(r)\right]\psi_{nl}\,d^3r.$$



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$$= -\frac{l}{2}\int f(r)\,|\psi|^2\,d^3r. \qquad -(8).$$

And $(\Delta E_{nl})_{fun} = \int \psi_{nl}^{*}\left[\left(\frac{l+1}{2}\right) f(r)\right]\psi_{nl}\,d^3r.$

$$= \frac{l+1}{2}\int f(r)\,|\psi_{nl}|^2\,d^3r. \qquad -(9).$$

The gap between the splitted energy levels is given by

$$\Delta(\Delta E_{nl}) = \left(\frac{l+1}{2} + \frac{l}{2}\right)\int f(r)\,|\psi_{nl}|^2\,d^3r. \qquad -(10).$$

Now we observe that.

(i) Corresponding to stretch case $j = (l+\frac{1}{2})$ the energy level is lower down while that for jacknife case $j=(l-\frac{1}{2})$ energy is raised.

(ii) The lowering & the raising corresponding to the two cases increase with the increase in the values of $l$.

(iii) And the most important observation is that gap between the splitted energy levels increases with $l$.

The sequence of energy levels is like shown in the figure. Spectroscopic notation is used to denote the terms. The letters (s, p, d, f etc) determine $l$ and the subscripts the $j$, the total angular momentum quantum no.

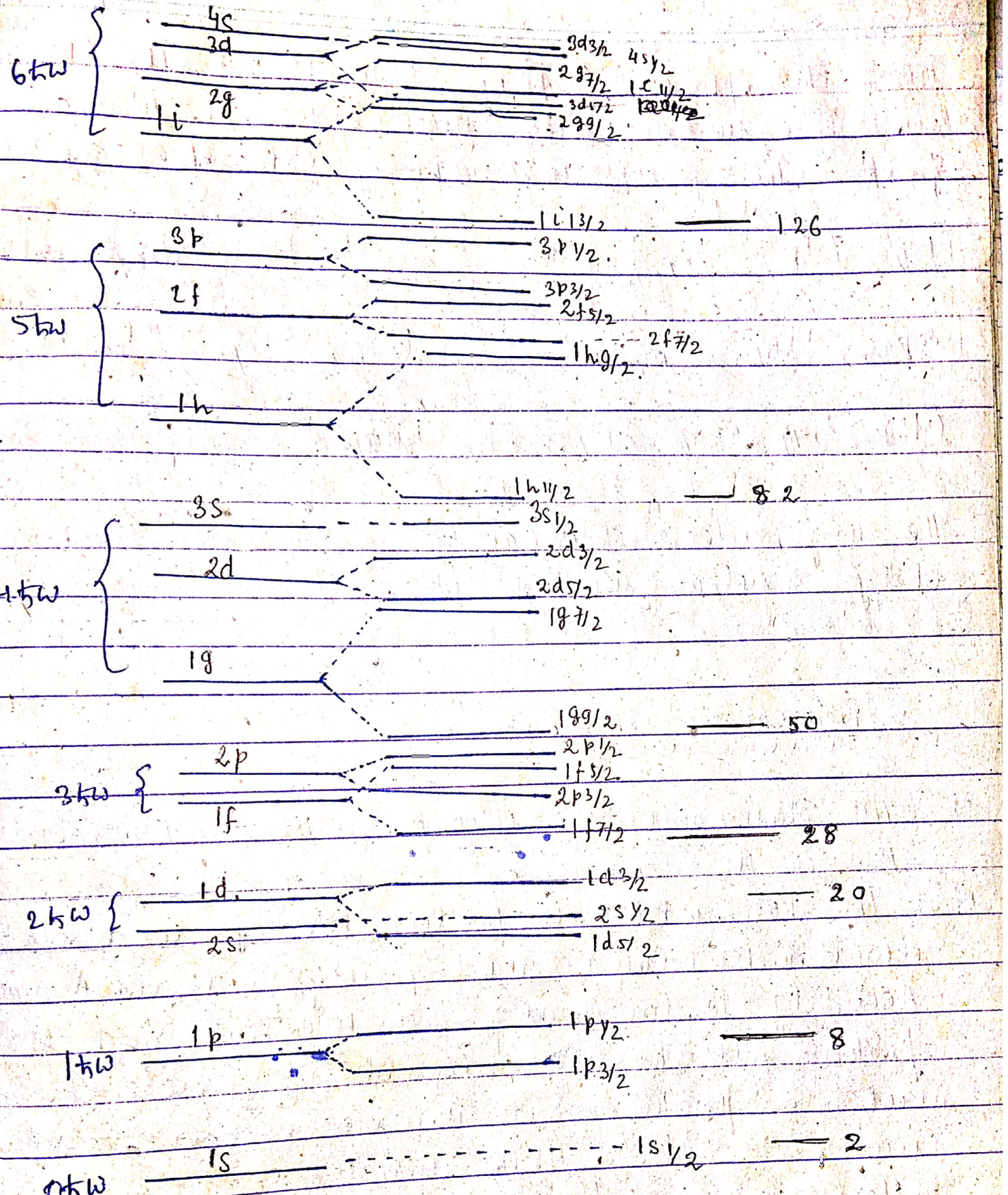
* $E_\Lambda = \hbar\omega\left(\Lambda + \frac{3}{2}\right), \quad \Lambda = 2n+l-2$

In fig in extreme left (i.e $0\hbar\omega$, $1\hbar\omega$ etc are difference in energy levels ie 1st is $\frac{3}{2}\hbar\omega$, 2nd $5/2\hbar\omega$ etc).

by using $\Lambda = 1$, n from 1, $l$ is found out and the splitting of states.



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Sequence of nuclear levels according to Shell model taking into account spin orbit interaction.


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Under the spin orbit Coupling scheme we get the energy levels of a nucleon in the nucleus to form several groups. These groups in ascending order of energy are:

$(1s_{1/2})$, $(1p_{3/2}, 1p_{1/2})$, $(1d_{5/2}, 2s_{1/2}, 1d_{3/2})$, $(1f_{7/2})$

$(2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2})$, $(1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2})$

$(1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2})$

The no of nucleons that can be accomodated in an energy level is given by $(2j+1)$.

## Predictions of Shell model.

i) The shell model correctly predicts the observed magic nos.

iii) The shell model can predict correctly the ground state angular momentum and parity of all odd A nuclei. According to this model the neutrons and the protons fill the respective energy levels. The no of neutron & proton that can be accomodated in a level is $(2j+1)$. As the neutron or proton fill the respective levels they get paired up with their angular momenta $\vec{J}$ antiparallel to each other giving rise to a net angular momentum = 0.

In case of odd A nuclei there remains one nucleon which is unpaired. The


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ground state angular momentum of an odd A nucleus is equal to the angular momenta (j value) of the last unpaired nucleon.

(iii) The shell model correctly predicts that ground state angular momentum of all Z even and N even nucleus to be zero.

(iv) The shell model correctly predicts the ground state parity of odd A nuclei. The parity of such a nucleus is given by $(-1)^l$. where l is the orbital angular momentum of the state occupied by the last unpaired nucleon.

## Limitations of Shell model

1. The shell model is incapable of predicting the ground state angular momenta of odd Z, odd N nuclei. This is because the model can not tell us how the angular momenta of the last unpaired neutron and the last unpaired proton combine with each other.
2. It also fails to explain large electric quadrapole moments and spheroidal shapes which many nuclei possess.
3. The ground states of the odd nuclei in range $150 \leq A \leq 190$ and at $A \geq 220$ can't be predicted by shell model.
4. It fails to explain magnetic moments of some other nuclei where the deviations are not so masked.
5. It also fails to explain excited states of even-even nuclei, the probabilities of radiative transition and nuclear Coulomb excitations.

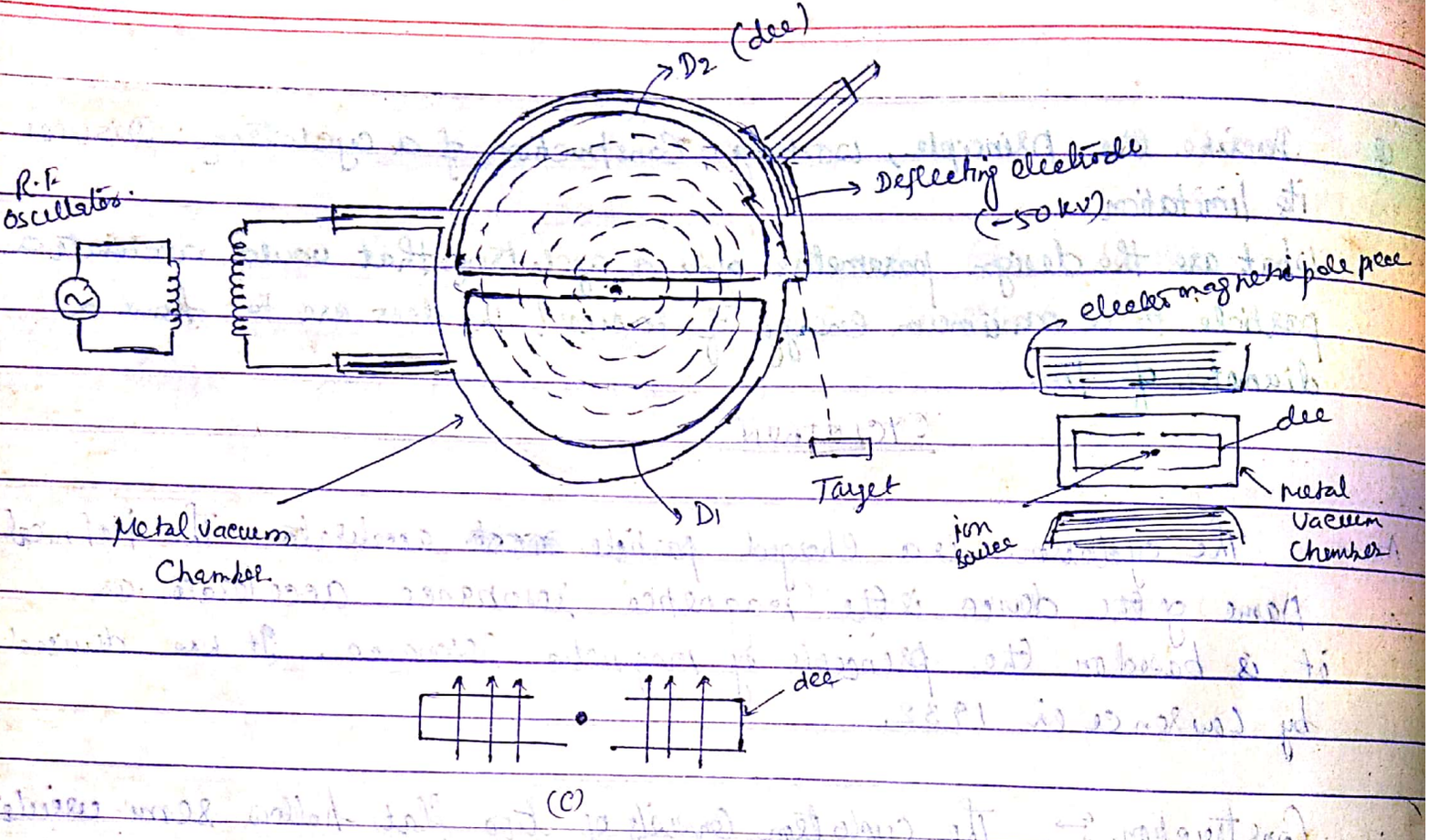

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# CYCLOTRON

→ The cyclotron is a charged particle accelerator. The technical name of the device is the 'magnetic resonance accelerator as it is based on the principle of magnetic resonance. It was devised by Lawrence in 1932.

Construction :→ The cyclotron consists of two flat, hollow semi-circular metal boxes called "dees" supported inside another metallic vacuum tank but insulated from it. The radius of the dees is large compared to their length. These dees are connected to a Radio-frequency voltage source. The metal box containing the dees is immersed in a high magnetic field of strength from 20-50 kilogauss produced by a large electromagnet, which acts perpendicular to flat faces of the dees. The positive ion source usually a capillary arc discharge type is placed at the centre of the dees. The pressure inside the metallic chamber is kept very low $\leq 10^{-6}$ mm of Hg to avoid the possibility of gaseous discharge within it. The accelerated ions are extracted out of the dees on to the target by means of an electrode near the dees and kept at large negative potential (-50 KV).


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Principle and working → In cyclotron the acceleration of the charged particles occurs in successive steps the energy gain being a fraction of the final energy achieved.

The positive ion when RF voltage is applied between the dees the positive ion from the ion source (protons, deuterons, or $\alpha$ particles) enter one of the dees (say $D_1$) when the phase of the RF voltage is such that they are attracted towards $D_1$ at the peak voltage $V_m$ and thereby gains the energy $qV_m$, q being the charge of the ion. Due to action of the transverse magnetic field these ions describe a semi circular path within $D_1$. Though the RF potential goes on changing during this time there is no further change in ion energy because the metallic dees acts as electrical shield. After describing a semicircle within $D_1$, as the ions reenter the


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narrow gap between the two dees they are attracted towards the opposite dee $D_2$ if the phase of the Rf voltage changes by 180°. So that $D_2$ attains a negative peak voltage w.r.t $D_1$ exactly at the same instant. The ions describe a semi circular path within $D_2$ of a slightly greater radius. As the ions emerge from $D_2$ and enter the gap between $D_1$ and $D_2$ they will gain an additional energy $qV_m$ in going from $D_2$ to $D_1$ if the phase of the Rf voltage changes by again by 180° exactly at that instant. Ultimately the ions gain quite high energy when they reach the periphery of the dees. In order that the ions may be accelerated to high energy by this method there must be resonance between the time for describing a semicircle within a dee and half the period of the Rf voltage ($T/2$)

Let $v$ be the velocity of an ion within a dee and $r$ the radius of curvature of its semicircular path within the dee, then for an ion of mass $M$ we can write $Bqv = \frac{Mv^2}{r}$ — (1)

or $\frac{r}{v} = \frac{M}{Bq}$ — (2)

So the time taken by an ion to describe a semi circle within the dee is $t = \frac{\pi r}{v} = \frac{\pi M}{Bq}$ — (3)

The time $t$ is independent of the radius $r$ of the ionic path & velocity $v$ of the ion. It depends only on the specific charge $q/M$ of the ion and is the same for the ions of same $q/M$. We assume $M$ does not change due to increasing velocity.

For resonance condition we have $t = T/2$

or $\frac{\pi M}{Bq} = \frac{T}{2} = \frac{1}{2f}$ where $f = 1/T$ the frequency of Rf voltage. — (4)


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& $f = \frac{Bq}{2\pi M}$ — (5)

The ion gain energy $qV_m$ at each D crossing.

If the ion cross the dee gap n times before reaching the periphery of the dees the final energy gained is $nqV_m$ If $v_m$ is the final velocity of the ion and R is the radius of the dees the maximum energy attained by the ions can be calculated from the relation

$$BqV_m = \frac{MV_m^2}{R} \quad \text{— (6)}$$

$$V_m = \frac{BqR}{M} \quad \text{— (7)}$$

final energy of ions is $E = \frac{1}{2}MV_m^2 = \frac{B^2q^2R^2}{2M} = 2\pi^2 M f^2 R^2$ — (8)

This shows final ion energy depends on the radius R of the dees and strength of the magnetic induction field B.

Also if V be the velocity in one ~~[illegible]~~ of the orbits of radius r. Then

$$BqV = \frac{MV^2}{r} \quad \& \quad V = \frac{Bqr}{M}$$

Energy of the ions have crossed n times $= nqV$

$$\& \quad \frac{1}{2}MV^2 = nqV \qquad [V \text{ is potential}]$$

$$\& \quad \frac{1}{2}M\left(\frac{Bqr}{M}\right)^2 = nqV$$

$$\& \quad \frac{1}{2}\frac{MB^2q^2r^2}{M^2} = nqV$$


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& $R = \frac{1}{B}\left(\frac{2MV}{q}\right)^{1/2} n^{1/2}$

Thus the successive orbits get closer as n becomes larger.

## Limitations of fixed frequency cyclotron.

Although the energy attained in a cyclotron increases with increasing radius of magnet used there is a practical limit which can to the energy which can be attained in a fixed frequency cyclotron due to relativistic increase of ion mass.

Writing $M = M_0/\sqrt{1-\beta^2}$ where $\beta = v/c$

& the eq $f = \frac{Bq}{2\pi M}$ becomes $f = \frac{Bq}{2\pi M_0}\sqrt{1-\beta^2}$

This limitation can be resolved in either of two ways →

so that the resonance condition is to obeyed

① the magnetic induction B is increased with time in such a way that the factor $\sqrt{1-\beta^2}$ cancels out, the frequency of rf field being constant

② the frequency f of the rf field is decreased with time in such a way that resonance condition holds at all instants the magnetic induction B remaining constant.


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